H

National
Qualifications
2025

# 2025 Mathematics

## Higher - Paper 1

## Question Paper Finalised Marking Instructions



SQA ©

**General marking principles for Higher Mathematics**

*Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.*

*For each question, the marking instructions are generally in two sections:*

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

*In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.*

**(a)** Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

**(b)** If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

**(c)** One mark is available for each •. There are no half marks.

**(d)** If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

**(e)** Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

**(f)** Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

**(g)** If an error is trivial, casual or insignificant, for example $6\times6=12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

**(h)** If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

This is a transcription error and so the mark is not awarded. (refers to $x-4x+3=0$)

This is no longer a solution of a quadratic equation, so the mark is not awarded. (refers to $x=1$)

The following example is an exception to the above

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

**(i) Horizontal/vertical marking**

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

| | $\bullet^5$ | $\bullet^6$ |
|---|---|---|
| $\bullet^5$ | $x = 2$ | $x = -4$ |
| $\bullet^6$ | $y = 5$ | $y = -7$ |

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

**(j)** In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

**(k)** Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

**(l)** Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

**(m)** In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

**(n)** You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

**(o)** You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

**(p)** If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

| | |
|---|---|
| Strategy 1 attempt 1 is worth 3 marks. | Strategy 2 attempt 1 is worth 1 mark. |
| Strategy 1 attempt 2 is worth 4 marks. | Strategy 2 attempt 2 is worth 5 marks. |
| From the attempts using strategy 1, the resultant mark would be 3. | From the attempts using strategy 2, the resultant mark would be 1. |

In this case, award 3 marks.

<u>Note: Marking from Image (MFI) annotation change from 2025</u>

A double cross-tick is used to indicate correct working which is irrelevant or insufficient to score any marks. In MFI marking instructions prior to 2025 this was shown as **ü**$_2$ or ü2.

From 2025, the double cross-tick will no longer be used in MFI. A cross or omission symbol will be used instead.

Marking Instructions for each question

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 1. | | | •1 find $y$-coordinate<br><br>•2 differentiate expression<br><br>•3 evaluate $\frac{dy}{dx}$ when $x=2$<br><br>•4 determine equation of tangent | •1 5<br><br>•2 $3x^2-4x$<br><br>•3 4<br><br>•4 $y=4x-3$ | 4 |

**Notes:**

1. Where candidates integrate, only •1 is available. For example, see Candidate F.
2. •3 is not available for $y=4$. However, where $y=4$ comes from substituting into $3x^2-4x$, or 4 is then used as the gradient of the straight line, •3 may be awarded. See Candidates A, B and C.
3. •4 is only available where candidates attempt to find the gradient by substituting $x=2$ into their derivative.
4. Where $x=2$ has not been used to determine the $y$-coordinate, •4 is not available.

**Commonly Observed Responses:**

**Candidate A - incorrect labels and no evidence of substitution**

$y=5$ — •1 ✓ - BoD

$y=3x^2-4x$ — •2 ✓

$y=4$ — •3 ✗

**Evidence for •4 would need to appear in the equation of the line for •3 to be awarded**

**Candidate B - incorrect labels and no evidence of substitution**

$y=5$ — •1 ✓ - BoD

$y=3x^2-4x$ — •2 ✓

$y=4$

$y=4x-3$ — •4 ✓ •3 ✓ - BoD

**Candidate C - incorrect labels with substitution**

$y=5$ — •1 ✓

$y=3x^2-4x$ — •2 ✓

$y=3(2)^2-4(2)=4$ — •3 ✓

**Candidate D - no evidence of substitution**

$y=5$ — •1 ✓

$\frac{dy}{dx}=3x^2-4x$ — •2 ✓

$m=4$ — •3 ✓

**Candidate E - equating to zero**

$y=5$ — •1 ✓

$\frac{dy}{dx}=3x^2-4x=0$ — •2 ✓

$3(2)^2-4(2)=0$

$m=4$ — •3 ✗

$y=4x-3$ — •4 ✓1

**Candidate F - appearance of '$+c$'**

$y=5$ — •1 ✓

$\frac{dy}{dx}=3x^2-4x+c$ — •2 ✗ •3 ✗ •4 ✗

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 2. | | | •1 find the midpoint of AB<br><br>•2 calculate gradient of AB<br><br>•3 state perpendicular gradient<br><br>•4 determine equation of perpendicular bisector | •1 (5,7)<br><br>•2 $\frac{3}{4}$<br><br>•3 $-\frac{4}{3}$ stated or implied by •4<br><br>•4 $3y=-4x+41$ | 4 |

**Notes:**

1. •2 may be awarded for $\frac{-6}{-8}$ or equivalent.
2. The gradient of the perpendicular bisector must appear in fully simplified form at the •3 or •4 stage for •4 to be awarded. See Candidate A.
3. •4 is only available as a consequence of using a perpendicular gradient and a midpoint.
4. At •4 accept $3y+4x=41$, $3y+4x-41=0$, $y=-\frac{4}{3}x+\frac{41}{3}$ or any other rearrangement of the equation where the constant terms have been simplified.

**Commonly Observed Responses:**

| Candidate A | | Candidate B - $m = m_{perp}$ | |
|---|---|---|---|
| (5,7) | •1 ✓ | (5,7) | •1 ✓ |
| $m=\frac{6}{8}$ | •2 ✓ | $m=\frac{3}{4}=-\frac{4}{3}$ | •2 ✓ •3 ✗ |
| $m_{\perp}=-\frac{8}{6}$ | •3 ✓ | $3y=-4x+41$ | •4 ✓1 - BOD |
| $6y=-8x+82$ | •4 ^ | However | |
| | | $m=\frac{3}{4}$ | •2 ✓ |
| | | $=-\frac{4}{3}$ | •3 ✓ - BOD |
| | | $3y=-4x+41$ | •4 ✓ |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 3. | | | •1 express first term in integrable form | •1 $12x^{-2}\ldots$ | 4 |
| | | | •2 integrate first term | •2 $\frac{12x^{-1}}{-1}$ | |
| | | | •3 integrate second term | •3 $\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ | |
| | | | •4 complete integration | •4 $-12x^{-1}+\frac{2}{3}x^{\frac{3}{2}}+c$ | |

**Notes:**

1. Do not penalise the appearance of '$+c$', missing integral signs or missing '$dx$' at •1.
2. •2 is only available for integrating a term with a negative power.
3. Do not penalise the appearance of an integral sign and/or $dx$ throughout.
4. Do not penalise the omission of '$+c$' at •2 or •3.
5. The '$+c$' must appear in the first line of working where coefficients of both terms are fully simplified. See Candidates C and D.
6. All coefficients must be simplified at •4 stage for •4 to be awarded.
7. •4 is not available within an invalid strategy, such as differentiation.
8. Accept $\frac{12}{-x}+\frac{2}{3}x^{\frac{3}{2}}+c$ for •4. However, do not accept $\frac{12}{-1x}+\frac{2}{3}x^{\frac{3}{2}}+c$.

**Commonly Observed Responses:**

**Candidate A - Integrating over two lines**

$12x^{-2}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ •1 ✓

$\frac{12x^{-1}}{-1}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ •2 ✓ •3 ✗

$-12x^{-1}+\frac{2x^{\frac{3}{2}}}{3}+c$ •4 ✓1

**Candidate B - error in integration**

$\int\left(12x^{-2}+x^{\frac{1}{2}}\right)dx$ •1 ✓

$\frac{12x^{-3}}{-3}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+c$ •2 ✗ •3 ✓

$-4x^{-3}+\frac{2}{3}x^{\frac{3}{2}}+c$ •4 ✓1

**Candidate C - integration incomplete at •4 stage**

$-12x^{-1}+\frac{2x^{\frac{3}{2}}}{3}$ •2 ✓ •3 ✓ •4 ^

$-\frac{12}{x}+\frac{2}{3}\sqrt{x^3}+c$

**Candidate D**

$\frac{12x^{-1}}{-1}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ •2 ✓ •3 ✓

$\frac{12x^{-1}}{-1}+\frac{2x^{\frac{3}{2}}}{3}+c$

$-\frac{12}{x}+\frac{2\sqrt[3]{x^2}}{3}+c$ •4 ✗

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 4. | | | •1 apply $m\log_3 x = \log_3 x^m$<br><br>•2 apply $\log_3 x + \log_3 y = \log_3 xy$<br><br>•3 evaluate expression | •1 $\log_3 2^3 \ldots$<br><br>•2 $\log_3\left(2^3 \times \frac{1}{24}\right)$<br><br>•3 $-1$ | 3 |

**Notes:**

1. Do not penalise the omission of the base of the logarithm at •1 or •2.
2. Do not penalise the omission of brackets at •2.
3. Correct answer with no working, award 0/3.
4. •3 is only available for evaluating the logarithm of a unitary fraction.
5. Each line of working must be equivalent to the line above within a valid strategy. However, see Candidates B and C.

**Commonly Observed Responses:**

**Candidate A - introducing a variable**

$\log_3 \frac{1}{3}$ — •1 ✓ •2 ✓

$3^x = \frac{1}{3}$

$x = -1$ — •3 ✓

**Candidate B**

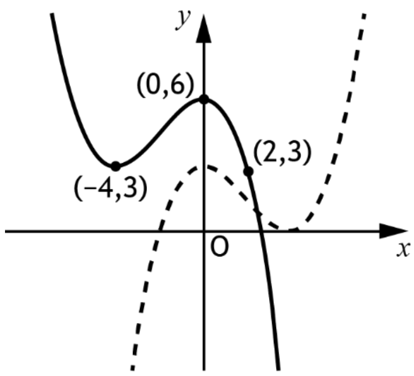
$3\log_3\left(2 \times \frac{1}{24}\right)$ — •2 ✗

$\log_3\left(2 \times \frac{1}{24}\right)^3$ — •1 ✓1 •3 ^

**Candidate C**

$\log_3\left(9 \times \frac{1}{24}\right)$ with no supporting working

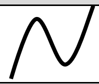
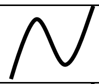
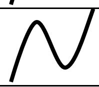
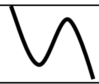
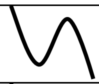
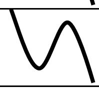
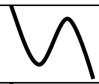
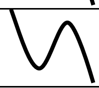
•1 ✗ •2 ✓1

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 5. | | | •[1] reflection in $y$-axis identifiable from graph<br><br>•[2] vertical translation of '$+3$' identifiable from graph | •[1,2]   | 2 |

**Notes:**

1. Where candidates do not sketch a cubic function, award 0/2.
2. Where only two of the three points are correctly transformed, award 0/2. However, see the table for possible exceptions.
3. Note that the position of the root, where shown, is not being assessed.
4. Ignore intersections (or lack of intersections) with the original graph.

**Commonly Observed Responses:**

| Function | Transformation of (−2,0) & (4,0) | Transformation of (0,3) | Shape | Award |
|---|---|---|---|---|
| Incorrect orientation | (2,3) & (−4,3) | (0,6) |  | 0/2 |
| $y = f(x) + 3$ | (−2,3) & (4,3) | (0,6) |  | 1/2 |
| $y = f(x) - 3$ | (−2,−3) & (4,−3) | (0,0) |  | 0/2 |
| $y = f(-x) - 3$ | (2,−3) & (−4,−3) | (0,0) |  | 1/2 |
| $y = -f(x) + 3$ | (−2,3) & (4,3) | (0,0) |  | 1/2 |
| $y = -f(x) - 3$ | (−2,−3) & (4,−3) | (0,−6) |  | 0/2 |
| Horizontal translation | (5,0) & (−1,0) | (3,3) |  | 1/2 |
| Horizontal translation | (−1,0) & (−7,0) | (−3,3) |  | 1/2 |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **6.** | (a) | (i) | •[1] determine $\sin q$ or $\cos q$ | •[1] $\sin q = \frac{1}{\sqrt{26}}$ OR $\cos q = \frac{5}{\sqrt{26}}$ stated or implied by •[2] | **3** |
| | | | •[2] substitute into formula for $\sin 2q$ | •[2] $2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}}$ | |
| | | | •[3] calculate $\sin 2q$ | •[3] $\frac{5}{13}$ | |
| | | (ii) | •[4] calculate $\cos 2q$ | •[4] $\frac{12}{13}$ | **1** |

**Notes:**

1. Evidence for •[1] may appear in (a)(ii).
2. Accept $q = \sin \triangleleft \left(\frac{1}{\sqrt{26}}\right)$, $q = \cos^{-1}\left(\frac{5}{\sqrt{26}}\right)$, $\sin\left(\frac{1}{\sqrt{26}}\right)$ or $\cos\left(\frac{5}{\sqrt{26}}\right)$ for •[1].
3. Where candidates substitute an incorrect value for $\sin q$ or $\cos q$, •[2] may be awarded if this value has previously been stated or it can be implied by a diagram or Pythagoras calculation. See Candidates C and D.
4. Where candidates explicitly state a value for $\sin q$ or $\cos q$, subsequent working must follow from that value for •[2] to be awarded.
5. •[3] is only available as a consequence of substituting into a valid formula at •[2].
6. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.
7. For $\sin 2q = \frac{1}{\sqrt{26}}$ and $\cos 2q = \frac{5}{\sqrt{26}}$ •[3] and •[4] are not available.

**Commonly Observed Responses:**

| **Candidate A** | | **Candidate B - unsimplified final answers** | |
|---|---|---|---|
| $2 \times \sin \frac{1}{\sqrt{26}} \times \cos \frac{5}{\sqrt{26}}$ | •[1] ✓ •[2] ✗ | $\sin q = \frac{1}{\sqrt{26}}$ | •[1] ✓ |
| $\frac{5}{13}$ | •[3] ✗ | $\sin 2q = 2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}}$ | •[2] ✓ |
| | | $\sin 2q = \frac{10}{26}$ | •[3] ^ |
| | | $\cos 2q = \frac{24}{26}$ | •[4] ✓[1] |
| | | $\sin(2q - r) = \frac{10}{26} \times \frac{4}{\sqrt{17}} - \frac{24}{26} \times \frac{1}{\sqrt{17}}$ | •[5] ✓ •[6] ✓ |
| | | $\sin(2q - r) = \frac{16}{26\sqrt{17}}$ | •[7] ✓[1] |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **6.** | (a) | (ii) | **(continued)** | | |

| **Candidate C - incorrect use of Pythagoras** | | **Candidate D - no evidence of Pythagoras** | |
|---|---|---|---|
| hypotenuse $= \sqrt{24}$ | $\bullet^1$ ✘ | $\sin 2q = 2 \times \frac{1}{\sqrt{24}} \times \frac{5}{\sqrt{24}}$ | $\bullet^1$ ^ $\bullet^2$ ✘ |
| $\sin 2q = 2 \times \frac{1}{\sqrt{24}} \times \frac{5}{\sqrt{24}}$ | $\bullet^2$ $✓_1$ | $\sin 2q = \frac{5}{12}$ | $\bullet^3$ $✓_1$ |
| $\sin 2q = \frac{5}{12}$ | $\bullet^3$ $✓_1$ | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 6. | (b) | | •[5] expand $\sin(2q-r)$<br><br>•[6] substitute into addition formula<br><br>•[7] evaluate $\sin(2q-r)$ | •[5] $\sin 2q\cos r-\cos 2q\sin r$<br><br>•[6] $\frac{5}{13}\times\frac{4}{\sqrt{17}}-\frac{12}{13}\times\frac{1}{\sqrt{17}}$<br><br>•[7] $\frac{8}{13\sqrt{17}}$ | 3 |

**Notes:**

8. Where candidates write $\sin\frac{5}{13}\times\cos\frac{4}{\sqrt{17}}-\cos\frac{12}{13}\times\sin\frac{1}{\sqrt{17}}$, award •[5]. However, •[6] and •[7] are unavailable.
9. •[6] and •[7] are only available as a consequence of substituting into a formula involving $\sin 2q$, $\cos 2q$, $\sin r$ and $\cos r$ from •[5].
10. For any attempt to use $\sin(2q-r)=\sin 2q-\sin r$, •[6] and •[7] are unavailable.
11. •[7] is only available for an answer expressed as a single fraction.

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 7. | (a) | | •1 use '−3' in synthetic division or in evaluation of a cubic<br><br>•2 complete division/evaluation and interpret result | •1 $\begin{array}{r\|rrrr} -3 & 5 & 16 & -1 & -12 \\ \hline & 5 & & & \end{array}$<br>**OR**<br>$5\times(-3)^3+16(-3)^2-(-3)-12$<br><br>•2 $\begin{array}{r\|rrrr} -3 & 5 & 16 & -1 & -12 \\ & & -15 & -3 & 12 \\ \hline & 5 & 1 & -4 & 0 \end{array}$<br>Remainder = 0<br>$\therefore (x+3)$ is a factor<br>**OR**<br>$f(-3)=0 \therefore (x+3)$ is a factor | 2 |

**Notes:**

1. Communication at •2 must be consistent with working at that stage – ie a candidate's working must arrive legitimately at 0 before •2 can be awarded.
2. Accept any of the following for •2.
   - '$f(-3)=0$ so $(x+3)$ is a factor'.
   - 'since remainder = 0, it is a factor'.
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow, \Rightarrow$ etc.
3. Do not accept any of the following for •2:
   - double underlining the '0' or boxing the '0' without a comment.
   - '$x=-3$ is a factor', '...is a root'.
   - the word 'factor' only with no link.

**Commonly Observed Responses:**

**Candidate A - grid method**

| | $5x^2$ | | |
|---|---|---|---|
| $x$ | $5x^3$ | $x^2$ | |
| 3 | $15x^2$ | | |

•1 ✓

| | $5x^2$ | $x$ | $-4$ |
|---|---|---|---|
| $x$ | $5x^3$ | $x^2$ | $-4x$ |
| 3 | $15x^2$ | $3x$ | $-12$ |

'with no remainder'

$\therefore (x+3)$ is a factor •2 ✓

**Candidate B - grid method**

| | $5x^2$ | | |
|---|---|---|---|
| $x$ | $5x^3$ | $x^2$ | |
| 3 | $15x^2$ | | |

•1 ✓

| | $5x^2$ | $x$ | $-4$ |
|---|---|---|---|
| $x$ | $5x^3$ | $x^2$ | $-4x$ |
| 3 | $15x^2$ | $3x$ | $-12$ |

$\therefore (x+3)(5x^2+x-4)=5x^3+16x^2-x-12$

$\therefore (x+3)$ is a factor •2 ✓

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 7. | (b) | | •3 state quadratic factor<br><br>•4 factorise quadratic<br><br>•5 state solutions | •3 $5x^2+x-4$ stated or implied by •4<br><br>•4 $(5x-4)(x+1)$<br><br>•5 $-3,\ \frac{4}{5},\ -1$ | 3 |

**Notes:**

4. The appearance of '$=0$' is not required for •5 to be awarded.
5. Candidates who identify a different initial factor and subsequent quadratic factor can gain •3, •4 and •5. See Candidate C.
6. •5 is only available as a result of a valid strategy at •3 and •4.
7. Accept $(-3,0)$, $(-1,0)$, $\left(\frac{4}{5},0\right)$ for •5.
8. $x=-3$ may appear in the working for part (a).

**Commonly Observed Responses:**

**Candidate C - starting again**

$(x+1)(5x^2+11x-12)$ •3 ✓

$(x+1)(5x-4)(x+3)$ •4 ✓

$x=-1,\ \frac{4}{5},\ -3$ •5 ✓

**Candidate D - alternative valid strategy**

| | | | | |
|---|---|---|---|---|
| −1 | 5 | 1 | −4 | •3 ✓ |
| | | −5 | 4 | |
| | 5 | −4 | 0 | •4 ✓ |

$x=-1,\ \frac{4}{5},\ -3$ •5 ✓

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 8. | | | **Method 1** | **Method 1** | 3 |
| | | | $\bullet^1$ rearrange and apply $\log_a x - \log_a y = \log_a \frac{x}{y}$ | $\bullet^1$ $\log_a \frac{75}{3} = 2$ | |
| | | | $\bullet^2$ write in exponential form | $\bullet^2$ $\frac{75}{3} = a^2$ stated or implied by $\bullet^3$ | |
| | | | $\bullet^3$ find $a$ | $\bullet^3$ 5 | |
| | | | **Method 2** | **Method 2** | |
| | | | $\bullet^1$ write 2 as $2\log_a a$ and apply $n\log_a x = \log_a x^n$ | $\bullet^1$ $\log_a 75 = \log_a a^2 + \log_a 3$ | |
| | | | $\bullet^2$ apply $\log_a x + \log_a y = \log_a xy$ | $\bullet^2$ $\log_a 75 = \log_a 3a^2$ | |
| | | | $\bullet^3$ find $a$ | $\bullet^3$ 5 | |

**Notes:**

1. Where candidates state expressions such as $\sqrt{25} = a^2$ or $a = 25^2$, $\bullet^3$ is not available.
2. Each line of working must be equivalent to the line above within a valid strategy.
3. Where candidates state $a = -5$ without discarding that 'solution', $\bullet^3$ is not available.
4. Do not penalise candidates who score out 'log' from equations of the form $\log_a x = \log_a y$. See Candidate H.
5. For the correct answer without working, award 0/3.

**Commonly Observed Responses:**

| **Candidate A - invalid strategy** | | **Candidate B - invalid strategy** | |
|---|---|---|---|
| $\log_a 75 = 2 + \log_a 3$ | | $\log_a 75 = 2 + \log_a 3$ | |
| $75 = a^2 + 3$ | $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$ ✗ | $\log_a 75 = \log_a 3^2$ | $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$ ✗ |
| $a = \sqrt{72}$ | | | |

| **Candidate C - valid strategy incorrectly applied** | | **Candidate D - valid strategy incorrectly applied** | |
|---|---|---|---|
| $\log_a 75 = 2 + \log_a 3$ | | $\log_a 75 = 2 + \log_a 3$ | |
| $\log_a (225) = 2$ | $\bullet^1$ ✗ | $\log_a \left(\frac{75}{3}\right) = 2$ | $\bullet^1$ ✓ |
| $225 = a^2$ | $\bullet^2$ ✓$_1$ | $\log_a 15 = 2$ | |
| $a = 15$ | $\bullet^3$ ✓$_1$ | $15 = a^2$ | $\bullet^2$ ✗ |
| | | $a = \sqrt{15}$ | $\bullet^3$ ✓$_1$ |

| **Candidate E** | | **Candidate F - BEWARE** | |
|---|---|---|---|
| $\log_a 75 = 2 + \log_a 3$ | | $\log_a 25 = 2$ | $\bullet^1$ ✓ |
| $\log_a \left(\frac{75}{3}\right) - 2$ | $\bullet^1$ ^ $\bullet^2$ ^ $\bullet^3$ ^ | $\log_a \sqrt{25}$ | $\bullet^2$ ✗ $\bullet^3$ ✗ |
| | | $\log_a 5$ | |
| | | $a = 5$ | SEEN |

| Question | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|
| 8. (continued) | | | |

| **Candidate G** | **Candidate H - scoring out** |
|---|---|
| $\log_a 75 = 2 + \log_a 3$<br>$\frac{\log_a 75}{\log_a 3} = 2$ •[1] ✗ •[2] ✗ •[3] ✗<br>$25 = a^2$<br>$a = 5$ | $\cancel{\log_a} 25 = \cancel{\log_a} a^2$<br>$a = 5$ •[3] ✓<br><br>**However**<br>$\frac{\cancel{\log_a} 25}{\cancel{\log_a}} = \frac{\cancel{\log_a} a^2}{\cancel{\log_a}}$<br>$a = 5$ •[3] ✗ |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 9. | | | •[1] substitute<br><br>•[2] write in standard quadratic form<br><br>•[3] find $x$-coordinates<br><br>•[4] find $y$-coordinates | •[1] $x^2+(x+1)^2-2x+6(x+1)-15(=0)$<br><br>•[2] $2x^2+6x-8=0$<br><br>•[3] •[4]<br>•[3] $-4,\ 1$<br><br>•[4] $-3,\ 2$ | 4 |

**Notes:**

1. •[2] is only available if '= 0' appears at the •[1] or •[2] stage.
2. Accept $x^2+3x-4=0$ for •[2].
3. The quadratic at •[2] must lead to two distinct real roots for •[3] and •[4] to be available.
4. Where candidates arrive at an equation which is not a quadratic at •[2] stage, then •[3] and •[4] are not available.
5. Where candidates identify **both** solutions by inspection, full marks may be awarded provided they justify that their points lie on **both** the line and the circle. Where candidates identify both solutions, but justify in only one equation, award 2/4.

**Commonly Observed Responses:**

**Candidate A - substituting for $x$**

$(y-1)^2+y^2-2(y-1)+6y-15=0$ •[1] ✓

$2y^2+2y-12=0$ •[2] ✓

$y=-3$ or $y=2$ •[3] ✓

$x=-4$ or $x=1$ •[4] ✓

**Candidate B - not squaring**

$x^2+(x+1)-2x+6(x+1)-15=0$ •[1] ✗ •[2] ✗

$x^2+5x-8=0$

$x=\frac{-5+\sqrt{57}}{2}$ and $\frac{-5-\sqrt{57}}{2}$ •[3] ✓[1]

$y=\frac{-3+\sqrt{57}}{2}$ and $\frac{-3-\sqrt{57}}{2}$ •[4] ✓[1]

The working for •[2] is not of equivalent difficulty

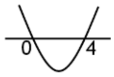
| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 10. | | | •[1] calculate $\lvert\mathbf{u}\rvert$<br><br>•[2] find expression for $\lvert\mathbf{v}\rvert$<br><br>•[3] evaluate $\mathbf{u}\cdot\mathbf{v}$<br><br>•[4] substitute into formula for scalar product<br><br>•[5] find $k$ | •[1] $\sqrt{2}$<br><br>•[2] $\sqrt{10+k^2}$<br><br>•[3] 4<br><br>•[4] $4=\sqrt{2}\times\sqrt{10+k^2}\times\cos 45°$ #<br><br>**OR**<br><br>$\cos 45° = \frac{4}{\sqrt{2}\times\sqrt{10+k^2}}$<br><br>•[5] $\sqrt{6}$ | 5 |

**Notes:**

1. Do not penalise the use of $\mathbf{a}$ and $\mathbf{b}$ in place of $\mathbf{u}$ and $\mathbf{v}$.
2. •[4] should not be awarded to candidates who simply state the formula $\cos 45° = \frac{\mathbf{u}\cdot\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}$.
3. Do not penalise the omission or incorrect use of units.
4. Where candidates find $\lvert\mathbf{v}\rvert = 4$ by substitution and equate that to $\sqrt{10+k^2}$, award •[3] and •[4].
5. •[5] is only available for a positive solution using a valid formula. However, do not penalise the appearance of $-\sqrt{6}$.

**Commonly Observed Responses:**

| | |
|---|---|
| **Candidate A - no reference to $\cos 45°$**<br>$4=\sqrt{2}\times\sqrt{10+k^2}\times\frac{1}{\sqrt{2}}$ •[1] ✓ •[2] ✓ •[3] ✓ •[4] ✓<br>$k=\sqrt{6}$ •[5] ✓ | **Candidate B - substitution followed by equating**<br>$\mathbf{u}\cdot\mathbf{v}=\sqrt{2}\times\sqrt{10+k^2}\times\cos 45°$ •[1] ✓ •[2] ✓<br>$\mathbf{u}\cdot\mathbf{v}=\sqrt{10+k^2}$<br>$\mathbf{u}.\mathbf{v}=4$ •[3] ✓<br>$4=\sqrt{10+k^2}$ •[4] ✓ |
| **Candidate C - all working contained in formula**<br>$4=\sqrt{1+1}\times\sqrt{1+3^2+k^2}\times\cos 45°$ •[3] ✓ •[4] ✓<br>$4=\sqrt{2}\times\sqrt{10+k^2}\times\cos 45°$ •[1] ✓ •[2] ✓ | **Candidate D - invalid notation**<br>$\mathbf{u}\cdot\mathbf{v}=\begin{pmatrix}1\\3\\0\end{pmatrix}$ •[3] ✗<br>$\mathbf{u}.\mathbf{v}=4$ |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 11. | | | •[1] use the discriminant | •[1] $(3k)^2 - 4(9)(k)$ | 4 |
| | | | •[2] apply condition | •[2] $(3k)^2 - 4(9)(k) > 0$ | |
| | | | •[3] identify roots of quadratic expression | •[3] 0, 4 | |
| | | | •[4] state range with justification | •[4] $k < 0$ and $k > 4$ with eg labelled sketch:  | |

**Notes:**

1. At •[1], treat the inconsistent use of brackets, for example $3k^2 - 4 \times 9 \times k$, as bad form only if the unbracketed terms are dealt with correctly in the next line of working. See also Candidate F.
2. Where candidates do not state an inequality in terms of $k$ but state expressions for $a$, $b$ and $c$, and $b^2 - 4ac > 0$, award •[2]. Where no expressions for $a$, $b$ and $c$ are stated, •[2] is only available where •[4] is awarded. See Candidates A, B and C.
3. If candidates have the condition 'discriminant < 0', 'discriminant ≤ 0' or 'discriminant ≥ 0', then •[2] is lost but •[3] and •[4] are available.
4. Ignore the appearance of $b^2 - 4ac = 0$ where the correct condition has subsequently been applied. See Candidate D.
5. If candidates only work with the condition 'discriminant = 0', then •[2] and •[4] are unavailable. See Candidate E.
6. Accept the appearance of 0 and 4 within inequalities for •[3].
7. For the appearance of $x$ in any expression of the discriminant, no further marks are available.

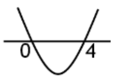
**Commonly Observed Responses:**

**Candidate A - no expressions for *a*, *b* and *c***

Two real and distinct roots $b^2 - 4ac > 0$

$9k^2 - 36k = 0$ •[1] ✓

$k = 0,\ k = 4$ •[3] ✓



$k < 0,\ k > 4$ •[2] ✓ •[4] ✓

**In this case •[2] is only available where •[4] is awarded**

**Candidate B - expressions for *a*, *b* and *c***

$a = 9,\ b = 3k,\ c = k$

$b^2 - 4ac > 0$ •[2] ✓

| Question | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|

**11.(continued)**

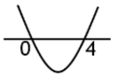
**Candidate C - working with ='0'**

$(3k)^2 - 4 \times 9 \times k$ $\bullet^1$ ✓

$9k^2 - 36k = 0$

$k = 0,\ k = 4$ $\bullet^3$ ✓

$b^2 - 4ac > 0$




$k < 0,\ k > 4$ $\bullet^2$ ✓ $\bullet^4$ ✓

In this case $\bullet^2$ is only available where $\bullet^4$ is awarded

**Candidate D**

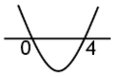
$(3k)^2 - 4 \times 9 \times k$ $\bullet^1$ ✓

$b^2 - 4ac = 0$

$9k^2 - 36k = 0$

$k = 0,\ k = 4$ $\bullet^3$ ✓

$9k^2 - 36k > 0$ $\bullet^2$ ✓



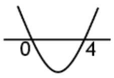

$k < 0,\ k > 4$ $\bullet^4$ ✓

**Candidate E - no inequality stated**

$(3k)^2 - 4 \times 9 \times k$ $\bullet^1$ ✓

$9k^2 - 36k = 0$ $\bullet^2$ ✗

$k = 0,\ k = 4$ $\bullet^3$ ✓

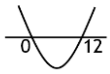



$k < 0,\ k > 4$ $\bullet^4$ ✗

**Candidate F - incorrect squaring**

$3k^2 - 4 \times 9 \times k > 0$ $\bullet^1$ ✗ $\bullet^2$ $✓_1$

$k = 0,\ k = 12$ $\bullet^3$ $✓_1$




$k < 0,\ k > 12$ $\bullet^4$ $✓_1$

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 12. | | | •1 integrate one term | •1 $6\sin x \ldots$ or $\ldots - \frac{8}{2}\cos 2x$ | 4 |
| | | | •2 complete integration | •2 $6\sin x - \frac{8}{2}\cos 2x + c$ | |
| | | | •3 substitute for $x$ and $y$ | •3 $4 = 6\sin\frac{\pi}{6} - \frac{8}{2}\cos\left(2\times\frac{\pi}{6}\right) + c$ | |
| | | | •4 state equation | •4 $y = 6\sin x - 4\cos 2x + 3$ stated explicitly | |

**Notes:**

1. Where candidates omit $+c$, only •1 is available.
2. Where candidates differentiate either term, •3 and •4 are not available.
3. Where candidates substitute into the original expression, •3 and •4 are not available.
4. Where candidates use an invalid approach, for example $6\cos x^2$, •3 and •4 are unavailable.
5. Do not penalise the appearance of an integral sign and/or '$dx$' at •1, •2 or •3.
6. Do not penalise candidates who substitute 30° after integrating.
7. •4 is only available where candidates work with a double angle after integrating.
8. Where candidates work with an expansion of $\cos 2x$, an explicit statement of $y = \ldots$ must still be stated. See Candidate D.

**Candidate A - incomplete substitution**

$y = 6\sin x - 4\cos 2x + c$ •1 ✓ •2 ✓

$y = 6\sin\frac{\pi}{6} - 4\cos\frac{2\pi}{6} + c$

$c = 3$ •3 ^

$y = 6\sin x - 4\cos 2x + 3$ •4 ✓1

**Candidate B - partial integration**

$y = 6\sin x + 8\sin 2x + c$ •1 ✓ •2 ✗

$4 = 6\sin\frac{\pi}{6} + 8\sin\frac{2\pi}{6} + c$ •3 ✓1

$c = 1 - 4\sqrt{3}$

$y = 6\sin x + 8\sin 2x + 1 - 4\sqrt{3}$ •4 ✓1

**Candidate C - integration incomplete at •2 stage**

$y = 6\sin x - 4\cos 2x$ •1 ✓ •2 ^

$y = 6\sin x - 4\cos 2x + c$

$4 = 6\sin\frac{\pi}{6} - 4\cos\frac{2\pi}{6} + c$ •3 ✓1

$y = 6\sin x - 4\cos 2x + 3$ •4 ✓1

**Candidate D - use of double angles**

$y = 6\sin x - 4\cos 2x + c$ •1 ✓ •2 ✓

$y = 8\sin^2 x + 6\sin x - 4 + c$

$4 = 8\sin^2\left(\frac{\pi}{6}\right) + 6\sin\left(\frac{\pi}{6}\right) - 4 + c$ •3 ✓

$c = 3$

$y = 8\sin^2 x + 6\sin x - 1$ •4 ✓

**All three expansions of $\cos 2x$ are valid.**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **13.** | (a) | | •1 equate derivative to zero and solve for $x$ | •1 $(x+5)(2-x)=0$ <br> **OR** <br> $f'(x)=0$ <br> **OR** <br> $\frac{dy}{dx}=0$ **stated explicitly** leading to −5 and 2 | **3** |
| | | | •2 construct nature table(s) | •2 | |
| | | | •3 interpret table and state conclusions | •3 min at $x=-5$; max at $x=2$ | |

| $x$ | ... | −5 | ... | 2 | ... |
|---|---|---|---|---|---|
| $f'(x)$ | − | 0 | + | 0 | − |
| (shape) | \ | _ | / | − | \ |

(•2 awarded for the $-5$ column, •3 for the $2$ column)

**Notes:**

1. •2 and •3 may be awarded vertically.
2. •3 is still available in cases where a candidates table of signs does not lead legitimately to a maximum/minimum shape.
3. Candidates may use the second derivative. See Candidates A and B.
4. Ignore any $y$-coordinates.
5. •2 is not available where any errors are made in calculating values of $f'(x)$.

**Commonly Observed Responses:**

| **Candidate A - second derivative** | **Candidate B - second derivative** |
|---|---|
| $f''(x)=-2x-3$ | $f''(x)=-2x-3$ |
| •2 ✓ $f''(-5)>0$ So min at $x=-5$ ; •3 ✓ $f''(2)<0$ so max at $x=2$ | •2 ✓ $f''(-5)=7>0$ so min at $x=-5$ ; •3 ✓ $f''(2)=-7<0$ so max at $x=2$ |
| •2 ✓ •3 ✓ | •2 ✓ •3 ✓ |

| Question | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|

**13.(a) (continued)**

For the table of signs for a derivative, **accept**:

| $x$ | $-5^-$ | $-5$ | $-5^+$ |
|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $+$ |
| Slope or shape | \ | — | / |

| $x$ | $\rightarrow$ | $-5$ | $\rightarrow$ |
|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $+$ |
| Slope or shape | \ | — | / |

Arrows are taken to mean 'in the neighbourhood of'

| $x$ | $a$ | $-5$ | $b$ |
|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $+$ |
| Slope or shape | \ | — | / |

where $a < -5$ and $-5 < b < 2$

**AND**

| $x$ | $2^-$ | $2$ | $2^+$ |
|---|---|---|---|
| $f'(x)$ | $+$ | $0$ | $-$ |
| Slope or shape | / | — | \ |

| $x$ | $\rightarrow$ | $2$ | $\rightarrow$ |
|---|---|---|---|
| $f'(x)$ | $+$ | $0$ | $-$ |
| Slope or shape | / | — | \ |

Arrows are taken to mean 'in the neighbourhood of'

| $x$ | $c$ | $2$ | $d$ |
|---|---|---|---|
| $f'(x)$ | $+$ | $0$ | $-$ |
| Slope or shape | / | — | \ |

where $-5 < c < 2$ and $d > 2$

For the table of signs for a derivative, **accept**:

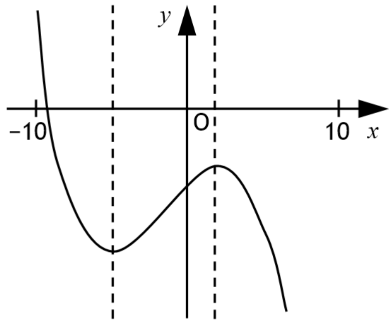
| $x$ | $\rightarrow$ | $-5$ | $\rightarrow$ | $2$ | $\rightarrow$ |
|---|---|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| Slope or shape | \ | — | / | — | \ |

Since the function is continuous $-5 \rightarrow 2$ is acceptable

| $x$ | $a$ | $-5$ | $b$ | $2$ | $c$ |
|---|---|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| Slope or shape | \ | — | / | — | \ |

Where $a < -5$, $-5 < b < 2$ and $c > 2$
Since the function is continuous $-5 < b < 2$ is acceptable

- For this question do not penalise the omission of '$x$' or the word 'shape'/'slope'.
- Stating values of $f'(x)$ is an acceptable alternative to writing '+' or '–' signs.
- Acceptable variations of $f'(x)$ are: $f'$, $\frac{df}{dx}$, $\frac{dy}{dx}$, $(x+5)(2-x)$ and $-x^2-3x+10$

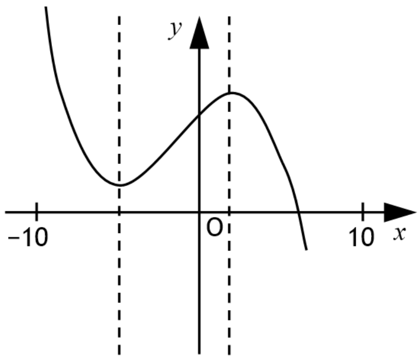
| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **13.** | (b) | | •[4] interpret result of (a) and first bullet point<br><br>•[5] interpret second bullet point within a cubic function<br><br>•[6] interpret third bullet point within a cubic function | •[4] cubic drawn with stationary points consistent with result of (a)<br><br>•[5] cubic drawn with $y$-intercept below the origin<br><br>•[6] cubic drawn with exactly one root; $-10 < \text{root} < 10$<br> | **3** |

**Notes:**

6. •[4] is awarded for turning points in a cubic graph consistent with the natures stated in part (a), or where no natures have been stated, the shape of the graph from the nature table.
7. Do not penalise the appearance of $y$-coordinates.
8. For a cubic graph which does not have two stationary points, award 0/3.
9. •[6] is not available where an extension of the graph of $y = f(x)$ will cross the $x$-axis.

**Commonly Observed Responses:**

**Candidate A – positive $y$-intercept**

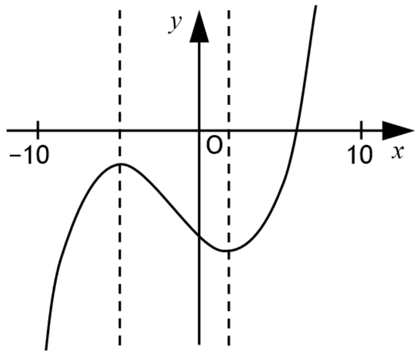



Award 2/3

**Candidate B – follow through from (a)**

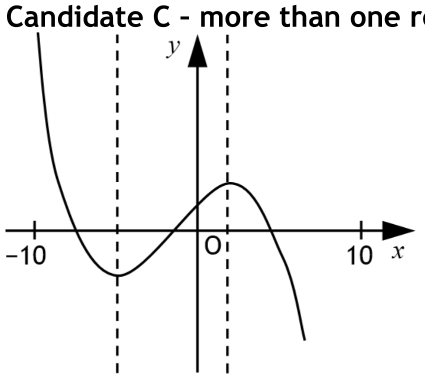
(a) max when $x = -5$ & min when $x = 2$

(b)



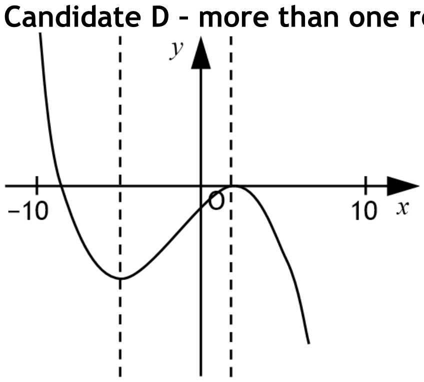

Award 3/3

**Candidate C – more than one root**



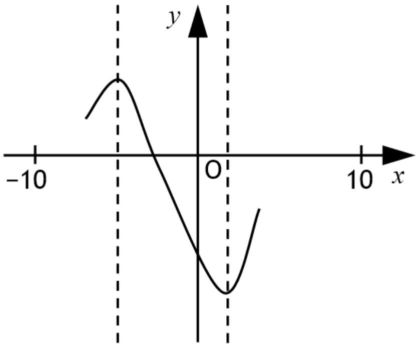
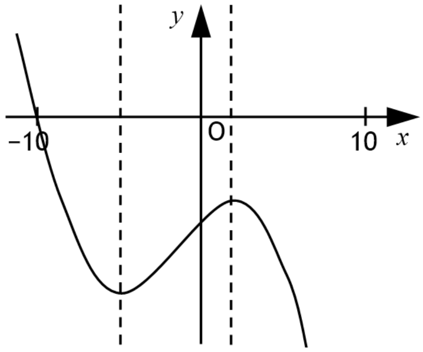

Award 1/3

**Candidate D – more than one root**




Award 2/3

| Question | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|
| **13.(b) (continued)** | | | |

| **Candidate E - more than one root implied** | **Candidate F - root outwith −10 < root < 10** |
|---|---|
|   Award 1/3 |   Award 2/3 |

**[END OF MARKING INSTRUCTIONS]**